Non-exponential Radiative Transfer for Volumetric Light Transport

Shivam Duggal
sduggal@andrew.cmu.edu
Carnegie Mellon University

Abstract

Radiative transfer framework governs the physics of light transport through a participating medium. The propagation of light through a medium can be affected via absorption, emission or scattering. Several works in computer graphics and computer vision focusing on forward/ inverse rendering have leveraged the radiative transfer framework for performing volumetric rendering of light/ radiance. General assumption made by these works is that the volume or the participating medium is either homogeneous or heterogeneous at macroscopic level, ignoring spatial correlations among the particles in the medium. The classical radiative transfer framework models the transmittance of light through such spatially-uncorrelated mediums as an exponential function. However, several real-world mediums like colloids (like milk, honey, soup), fog, smoke, clouds etc contain correlated particles/ scatters. Under such participating mediums, the transmittance function is no longer exponential. Through this project, we explore the affect of spatially correlated participating mediums on the radiative transfer framework. We showcase the effect of non-exponential transmittance on volumetric rendering by using: (a) parametric models for directly sampling the transmittance/ free-flight distributions (phenomenological sampling), (b) parametric models for sampling the extinction coefficients (physically-plausible sampling).

1 Background

The classical rendering equation ([7]) models the transport of radiance through a scene composed of several surfaces (with associated materials) and vacuum as the participating medium. Since the participating medium is vacuum, the radiance along a direction changes only on surface interactions.

More formally, the rendering equation is defined as:

$$L_o(x, \omega_o) = L_e(x, w_o) + \int_{\Omega} f_r(x, \omega_i, \omega_o) L_i(x, \omega_i) \langle \omega_i, n \rangle d_i \tag{1}$$

i.e the outgoing radiance $(L_o(x, \omega_o))$ at a point x in the direction (ω_o) is equal to the radiance emitted $(L_e(x, w_o))$ from point x in the direction (ω_o) plus the fraction of the incoming radiance scattered $L_i(x, \omega_i)$ from the point x along the direction (w_o). $f_r(x, \omega_i, \omega_o)$ refers to the bi-directional surface scattering term (BSDF).

When the participating medium is not vacuum, radiance between any two non-surface points (points in the medium) is also not constant. Light or radiance can be gained/ lost as it travels through a participating medium based on its interaction with the particles of the medium.

As shown in Figure 1, light can be absorbed, scattered (in-scattered or out-scattered) or emitted as it interacts with the medium. The radiative transfer equation (RTE) [5] is the equation for representing these interactions of light with the participating medium.

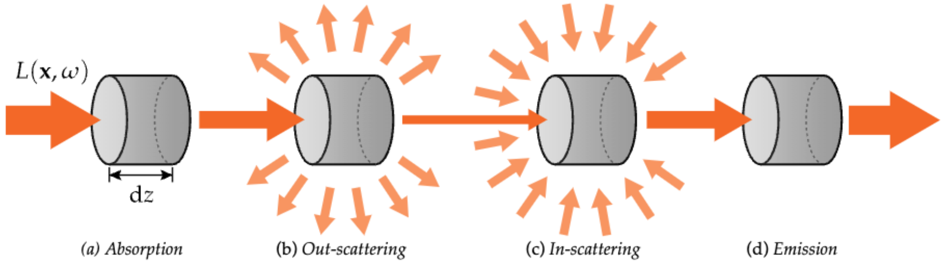

Figure 1: **Phenomenons of light modelled by Radiative Transfer Equation**: (a) Absorption, (b) Out-scattering lead to radiance loss, (c) In-scattering, (d) Emission lead to radiance gain

The radiative transfer equation defining the light transport only inside the particle medium is defined as:

$$\omega.\nabla L(x,\omega) = -\sigma_a(x)L(x,\omega) - \sigma_s(x)L(x,\omega) + \sigma_s(x)L_s(x,\omega) + \sigma_a(x)L_e(x,\omega) \tag{2}$$

where the first two terms represent the loss of radiance coming into the differential volume, and the last two terms represent the gain (radiance gained through scattering $(L_s(x,\omega) = f_p(\omega,\omega_i)L_i(x,\omega_i)d\omega_i)$ and emission inside the medium $(L_e(x,\omega))$. σ_s, σ_a are the extinction coefficients. The extinction coefficients depend on the density of the particles and the surface area, and hence directly represent the particle distribution. The left hand side of the RTE represent the directional derivative of the radiance. Please refer to Arvo *et al.* [3] for detailed derivation for the RTE.

Integrating both side of the Eq. 2, we get:

$$L(x,\omega) = \int_0^\infty Tr(x,y)\Big[\sigma_a(y)L_e(y,\omega) + \sigma_s(y)L_s(y,\omega)\Big] \tag{3}$$

Eq. 3 basically represents the total radiance received at point x (along direction ω) from all possible emitters/ scatters within the medium. The loss of light (i.e attenuation) as it travels between any two points (x, y) is given by the transmittance function $(Tr(x, y))$. Mathematically transmittance is the ratio of the outgoing radiance to the incoming radiance (also known as fractional visibility).

When the scene contains both the surfaces (at location z) and the participating medium (from 0 to z), Eq. 3 changes to:

$$L(x,\omega) = \int_0^z Tr(x,y)\Big[\sigma_a(y)L_e(y,\omega) + \sigma_s(y)L_s(y,\omega)\Big] + Tr(x,z)L(z,\omega) \tag{4}$$

$L(z,\omega)$ represents the radiance from the radiance coming out of the surface at location z and is written as a combination of light emitted from the surface and light scatter from the surface (function of BSDF):

$$L(z,\omega) = L_e(z,\omega) + \int_\Omega f_r(z,\omega,\omega_i)L_i(z,.\omega_i)\langle n(z),\omega_i\rangle d\omega_i \tag{5}$$

The equation (Eq. 5) is known as the volumetric rendering equation (Figure 2). We use this volume rendering form for monte carlo light transport [10] in our project.

1.1 Exponential Radiative Transfer

The classical radiative transfer equation [5] assumes the participating medium to be composed of independent and uncorrelated particles (i.e particle distribution is modelled as a poisson distribution). Under such phenomenon, the transmittance function is an exponential function, and is defined as:

$$Tr(x, x_t) = exp(-\tau(x, x_t))$$

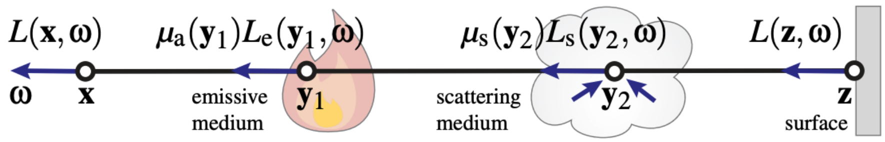

Figure 2: **Volumetric Rendering Equation**: As light travels from a surface and through a medium, it gets scattered, absorbed and emitted. μ in the figure is same as σ in the text in background section.

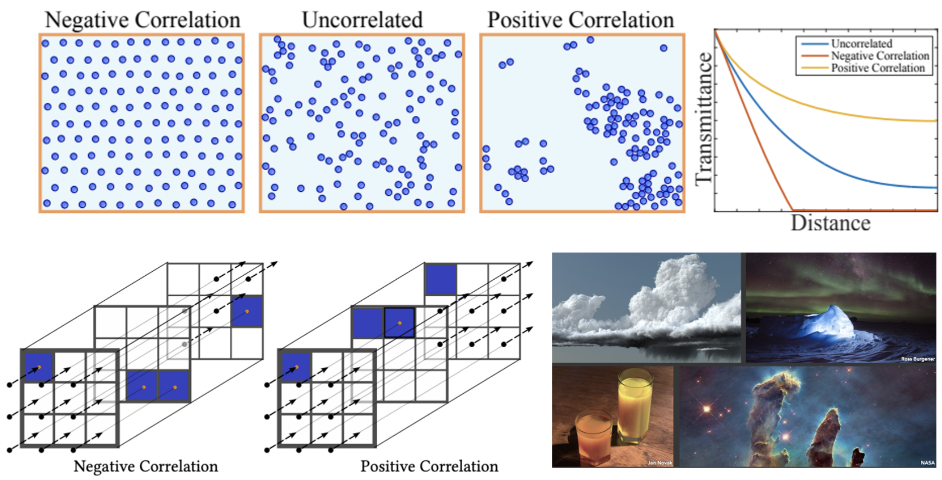

Figure 3: **Participating medium scatterer correlations**: While classical RTE assumes uncorrelated particles, several material (bottom right) have correlated particles leading to non-exponential transmittance (top row).

$$\tau(x, x_t) = \int_0^t \sigma_t(x_s) ds$$

where $\sigma_t = \sigma_a + \sigma_s$. For homogeneous medium, σ_t is same for all points and hence $\tau(x, x_t) = t\sigma_t$

This exponential form of the transmittance is extracted from the Beer-Lambert's law.

1.2 Non-exponential Radiative transfer

Exponential transmittance is a result of the spatial-independence of the particles of the participating medium. However, many real-world mediums (Figure 3 bottom right) have spatially correlated particles (like atmospheric clouds, milk etc). The particle correlation can be positive (Figure 3 bottom middle) or negative (Figure 3 bottom left). Positive correlation leads to cluster of particles and hence leads to larger transmittance of light. On the other hand, negative correlation leads to decreased transmittance (Figure 3 top).

Non-exponential transmittance has been tackled by several prior works though: Generalized Boltzmann Equation (GBE) by Larsen [8], Generalized Linear Boltzmann Equation by Larsen and Vasques (GLBE) [9], Extended Generalized Boltzmann Equation by Jarabo *et al.* [6].

In this project, we follow the formulation of Bitterli *et al.* [4], who represent transmittance though a spatially-correlated medium as average/ ensemble behavior of different light paths interacting with a stochastic medium. In other words, each light path is considered in a separate exponential medium, and the total transmittance of light through the medium equals the stochastic average of transmittance of different light paths. In this project, we use Bitterli *et al.* [4]'s formulation of non-exponential transmittance.

1.3 Non-exponential framework of Bitterli *et al.*

Bitterli *et al.* [4] model transmittance between two points in a spatially correlated medium ($Tr(x_i, x_t)$) as the stochastic ensemble over multiple light paths (or realizations μ):

$$Tr(x, x_t) = \langle Tr_\mu(x, x_t) \rangle$$

where different μ represent different medium for different light paths, and $\langle f_\mu(x) \rangle = \int_R f_\mu(x) dP(\mu)$ represent the stochastic ensemble of any function f.

Ideally, ensemble has to be taken over the path measurement function ([11] thesis chapter 8). But by making the assumption that the phase function, albedo, sensor and source response functions are uncorrelated with the realization (μ), we can simply focus on taking the ensemble over the free-flight distributions (free-flight distribution = transmittance * extinction coefficient of the end point) of the different light paths. Furthermore, the free-flight distribution ($T(x, x_k)$) of the entire path can be broken down into product of the free-flight distributions of the sub-paths.

$$T(x, x_k) = \prod_{i=0}^{k-1} T(x_i, x_{i+1}) = \langle \prod_{i=0}^{k-1} T_\mu(x_i, x_{i+1}) \rangle = \langle \prod_{i=0}^{k-1} Tr_\mu(x_i, x_{i+1}) \sigma_\mu(x_{i+1}) \rangle$$

Like Larsen and Vasques [9], Bitterli *et al.* further make the assumption that transmittance function is a short lived i.e. transmittance only depends on the distance to the last interaction point along the path. As a result of this assumption, we can now take ensemble over each sub-path instead of ensemble over each full path:

$$\langle \prod_{i=0}^{k-1} T_\mu(x_i, x_{i+1}) \rangle = \prod_{i=0}^{k-1} \langle T_\mu(x_i, x_{i+1}) \rangle = \prod_{i=0}^{k-1} \langle Tr_\mu(x_i, x_{i+1}) \sigma_\mu(x_{i+1}) \rangle$$

Based on the end points of the sub-path (surface or medium), the free-flight distribution might have either of the two terms: $\sigma_\mu(x_i)$ and $\sigma_\mu(x_{i+1})$. If both the end-points lie on the medium, then free-flight would have both the $\sigma_\mu(x_i)$ and $\sigma_\mu(x_{i+1})$ term, and it will not have any of the two, when both points are on the surface (reducing down to just transmittance). Thus, there would exist 4 different formulations of the free-flight terms based on the location of the end-points (on medium or on surface). Figure 4 represents these 4 free-flight distributions and the relationship between them.

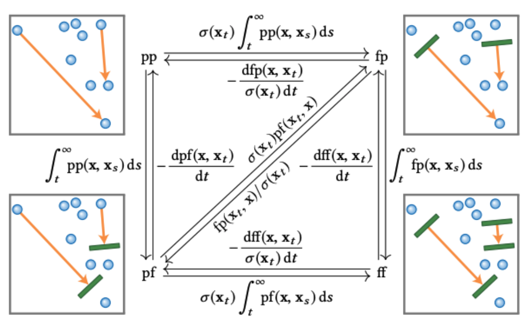

Figure 4: Four possible representations of the free-flight distribution

2 Method

Bitterli *et al.* are based on the monte carlo path integral formulation of light transport. Instead we utilize the monte carlo version of the above mentioned volume rendering/ tracing framework.

We used the next-event estimation based volumetric tracer of the DIRT renderer (used in the course [1]). As an advanced/ intermediate feature (6 points), we adapt the non-exponential radiative transfer framework of Bitterli *et al.* to DIRT's volume tracer. This involved the following operations:

- Created a transmittance base class, which invokes all the difference free-flight distributions based on the location of the end points.
- Created a sub-class for each experimented transmittance function (eg: LinearTransmittance, ExponentialTransmittance etc).

Figure 5: **Background scene elements:** We simply used random RGB images from the internet as background element texture maps. (Left) Wall texture map, (Right) Floor texture map.

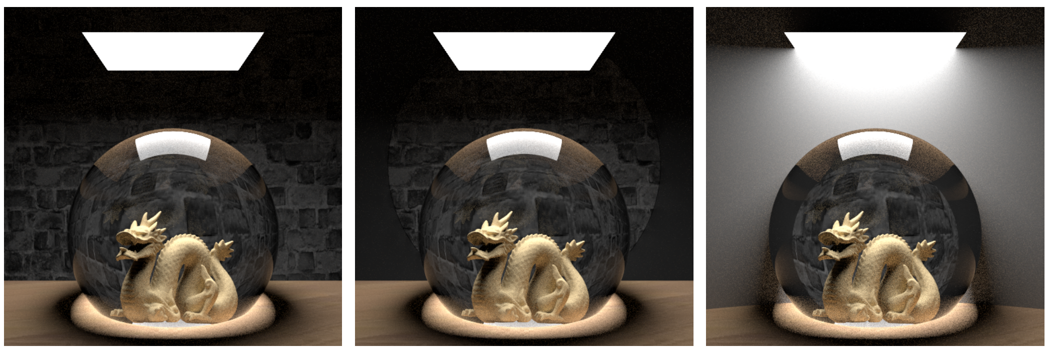

Figure 6: **Light attenuation modelled by varying the slope of linear transmittance function**: We model the scene as a homogeneous medium, and represent the transmittance function (ff) as a parametric linear function ($\text{ff}(x) = max(0, 1 - \frac{x}{b})$). From left to right, we decrease the value of parameter b (from 1000 to 1.72 to 0.8) and hence decrease the visibility and increase the blurriness

.

- Updated DIRT's medium class to invoke different transmittance classes (depending upon a user defined json paramrter) rather than simply using exponential transmittance.

In order to determine the four different free-flight distributions, we adopted two different sampling strategies: (a) Phenomenological representation of free-flight using analytical PDFs (Section 3.2), (b) Closed-form transmittance via direct sampling of average extinction coefficient (Section 3.3). We elaborate on the two different strategies in the experiments section.

3 Experiments

In this section, we first define our scene setup (used in [1]'s rendering competition), and then define the two different strategies we used to generate non-exponential transmittance and free-flight distributions. In both the experiments, we assume the participating medium to be homogeneous.

3.1 Scene Setup

We construct a scene consisting of a greyish stone wall (Figure 5 left) and a brown wooden floor (Figure 5 right) as the background. The foreground is composed of the golden Stanford Dragon [2] placed inside a dielectric lobe (refractive index = 1.4). An area light source is placed on the ceiling. The participating medium is homogeneous i.e. the extinction coefficients do not change spatially within the medium[1].

[1]A homogeneous medium can also be spatially-correlated. Homogeneous spatial-correlation just means that the particle spatial correlations are statistically same throughout the medium

Figure 7: **Phenomenological scene generation using analytical sampling of transmittance/ free-flight distributions:** (top left) exponential transmittance, (top right) constant transmittance or vacuum, (bottom left) linear transmittance (intercept=1.72), (bottom right) quadratic transmittance

3.2 Transmittance via directly sampled free-flight PDFs

[6 points]

In this experiment, we analytically chose the transmittance function (ff in Figure 4) to be one of exponential, linear, constant or quadratic functions. Then based on the formulation of transmittance function (ff), we computed the value of extinction coefficient ($\sigma = \text{ff}'(0)$), and then derived the other free-flight distributions (pf, fp, pp) using the formulation shown in Figure 4. Computing the free-flight distributions requires performing PDF, CDF and inverse CDF computations. The computed ff, pf, pp, fp distributions were then implemented in their corresponding transmittance child class. Figure 7 showcase the comparison of the images rendered by using different analytical distributions as the transmittance function (ff). As intuitive, constant transmittance (similar to having no medium [7]) leads to the entire scene being clearly visible. Linear transmittance ($\text{ff}(x) = max(0, 1 - \frac{x}{b})$) leads to linear decrease in the scene visibility with the light path length, while quadratic transmittance leads

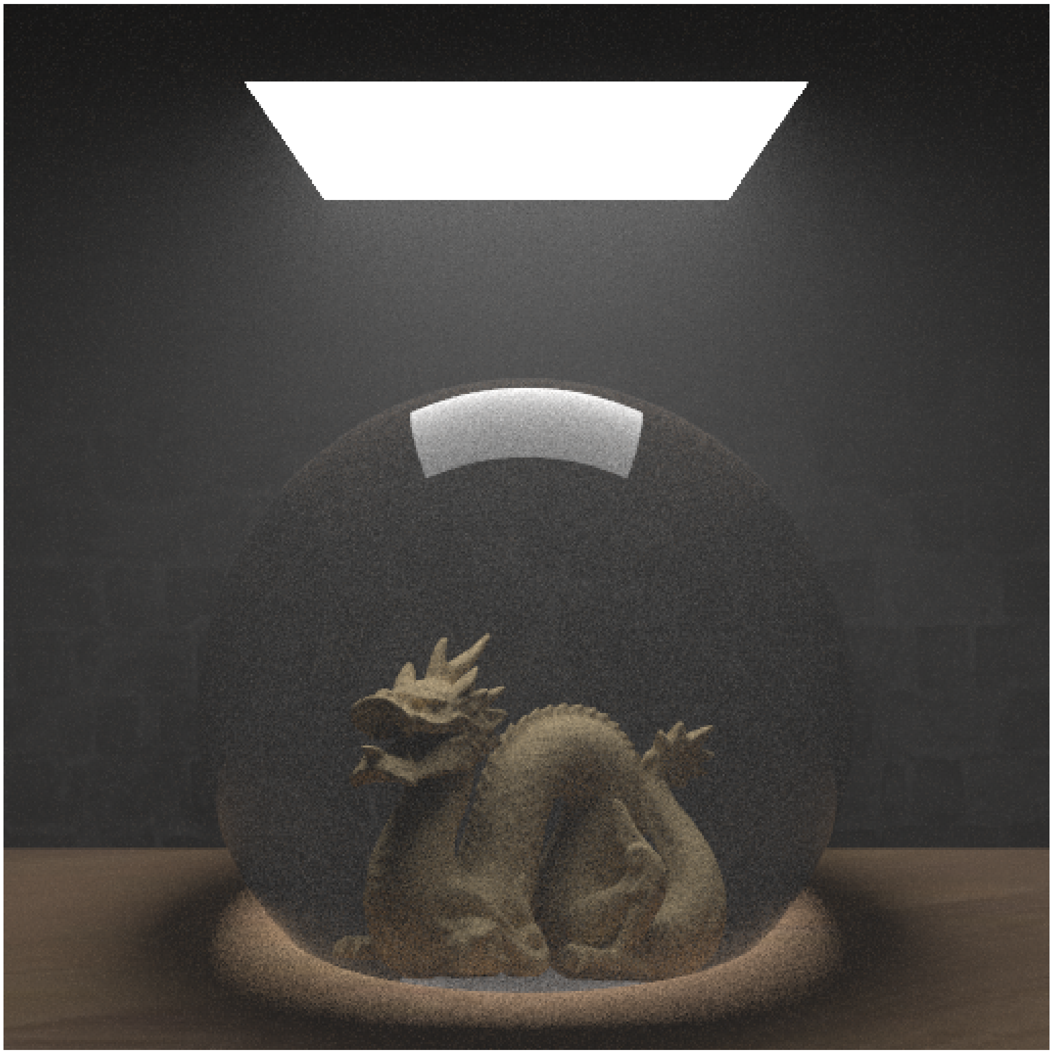

Figure 8: **Image generated using closed form generation of transmittance using analytical sampling of extinction coefficient distribution**

to quadratic decrease in the scene visibility with the light path length (hence being more blurred). From Figure 6, we see that as we increase the linear fall-off of the transmittance (i.e decrease the parameter b of ff), the background at larger depth starts becoming blurry or black.

3.3 Closed-form transmittance via direct sampling of average extinction coefficient

[6 points]

To derive a physically-plausible representation of the participating medium, its important to either directly generate the particles of the scene (intractable or expensive way) or directly sample the average extinction coefficient distribution analytically. We used a gamma distribution as our analytic extinction coefficient distribution and generated the free-flight distribution from it in closed form (using characteristic function equation). Please refer to Bitterli *et al.* [4] (Section 4.4) for more details. Figure 8 showcase the rendering of the scene generated using this methodology.

4 Acknowledgement & Credits

We would like to thank Prof. Ioannis Gkioulekas for being an amazing instructor of an amazing course [1], and also for resolving all queries/ questions regarding light transport throughout the course.

We would also like to thank Ryan Po and Bailey Miller for resolving DIRT related doubts, and thank Bailey Miller for several fruitful discussions (specially the one on non-exponential RTE).

References

[1] http://graphics.cs.cmu.edu/courses/15-468/.

[2] http://graphics.stanford.edu/data/3dscanrep/.

[3] James Arvo. Transfer equations in global illumination. 1993.

[4] Benedikt Bitterli, Srinath Ravichandran, Thomas Müller, Magnus Wrenninge, Jan Novák, Steve Marschner, and Wojciech Jarosz. A radiative transfer framework for non-exponential media. *ACM Transactions on Graphics (Proceedings of SIGGRAPH Asia)*, 2018.

[5] Subrahmanyan Chandrasekhar. *Radiative transfer*. 1960.

[6] Adrian Jarabo, Carlos Aliaga, and Diego Gutierrez. A radiative transfer framework for spatially-correlated materials. 2018.

[7] James T. Kajiya. The rendering equation. In *Computer Graphics*, 1986.

[8] Edward W. Larsen. A generalized boltzmann equation for "non-classical" particle transport. 2007.

[9] Edward W. Larsen and Richard Vasques. A generalized linear boltzmann equation for non-classical particle transport. 2011.

[10] Jan Novák, Iliyan Georgiev, Johannes Hanika, and Wojciech Jarosz. Monte Carlo methods for volumetric light transport simulation. *Computer Graphics Forum (Proceedings of Eurographics - State of the Art Reports)*, 2018.

[11] Eric Veach. *Robust Monte Carlo Methods for Light Transport Simulation*. PhD thesis, 1998.